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What is physics problem solving competency? The views of Arnold Sommerfeld and Enrico Fermi

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# What is physics problem solving competency?

## The views of Arnold Sommerfeld and Enrico Fermi

### Abstract

A central goal of physics education is to teach problem-solving competency, but the nature of this competency is not well-described in the literature. The present article uses recent historical scholarship on Arnold Sommerfeld and Enrico Fermi to identify and characterize two positions on the nature of physics problem-solving competency. The first, Sommerfeld's, is a "theory first, phenomenon second" approach. Here the relevant problems originate in one of the theories of physics and the goal of the problem-solver is to make a mathematical analysis of the suitable equation(s) and then give a qualitative analysis of the phenomenon that arise from these mathematical results. Fermi's position is a "phenomenon first, theory second" approach, where the starting point is a physical phenomenon that is analyzed and then brought into the realm of a physics theory. The two positions are illustrated with solutions to two problems and it is shown that the two positions are reflected in problem collections of university educations in physics.

Keywords: Problem Solving Competency, Physics Education, History of Physics, Philosophy of Physics

### 1. Introduction

Problem-solving plays an extensive role in the physics curriculum on most educational levels. Two types of justification are given for this situation. The first type focuses on how problem-solving may facilitate students' learning of physics, e.g. of concepts. The other type sees the development of

25 students' problem-solving competency in itself as a goal of physics education (e.g., Rigden 1987;  
26 Mestre et al. 1993; Maloney 1994; Hsu et al. 2004, Gerace and Beaty 2005; Walsh et al. 2008;  
27 Maloney 2011). An argument for the latter view is that physics education (at least at the higher  
28 educational levels) ought to reflect the nature of physics as a scientific discipline and problem-  
29 solving is an important activity of scientific inquiry in general (Laudan 1977), and in physics in  
30 particular. The proponents of this view also argue that physics is in a unique position to help  
31 students develop the required skills for solving genuine problems of the real world (Rigden 1987),  
32 so physics have something to offer in this respect. While the two types of justification for problem-  
33 solving do not necessarily collide, they do represent two different orientations regarding what are  
34 the goals and what are the means. Moreover, due to their different aims, the two types often have  
35 different views on which problems are relevant for physics education.

36 In the present article, we focus on the development of problem-solving competency as an  
37 instructional goal in itself at the university/college level. The question we ask is: what is the nature  
38 of physics problem-solving competency? The answer to this question is important because students  
39 and teachers alike need to be aware of this nature as their beliefs impact how they deal with  
40 problem-solving in teaching situations: Mason & Singh (2010) pointed out that students' attitudes  
41 and approaches to problem-solving in physics can profoundly affect their development of physics  
42 expertise, including problem-solving competency. Ding (2014) argued that physics teachers' views  
43 of problem-solving influence their teaching: "The interplay between faculty's views of problem-  
44 solving and their choice of related activities can influence the conceptualization, design, and  
45 implementation of these introductory courses, thus having far-reaching implications for higher  
46 education." (Ding, 2014, p. 137) More generally, being an educational goal in itself, the specific  
47 nature of this competency should influence the choice of tasks, pedagogical methods, assessment  
48 strategies etc. that are employed in educational settings. Consequently, physics teachers and  
49 students should understand the nature of physics problem-solving competency and therefore we  
50 need to have an understanding of this nature.

51

52           Unfortunately, the description of the nature of physics problem-solving competency is left  
53 either implicit or somewhat fragmentary in the physics education literature. Policy documents on  
54 physics education, such as Beneitone et al. (2007), Tuning Project (2007), The Quality Assurance  
55 Agency for Higher Education (2017), state that university/college students should learn physics  
56 problem-solving competency, which the Latin-American Tuning Project characterized as “The  
57 capacity to pose, analyse and solve physical problems, both theoretical and experimental, through  
58 the use of numerical, analytical or experimental methods.” (Beneitone et al., 2007, p. 155). Physics  
59 education researchers and physics educators, e.g., Walsh, Howard, Bowe, (2007), Gerace and  
60 Beatty (2005), Heron & Meltzer (2005), Knight (2004), embrace this goal: “one of the principal  
61 goals of a physics course is to produce adept problem solvers who can transfer their knowledge and  
62 understanding to real world situations” (Walsh et al. 2007, p. 020108-1). So, these documents take a  
63 somewhat holistic approach and state that physics students should be able to tackle problems in  
64 physics. In effect, they take competent problem-solving to be the ability to solve physics problems,  
65 so the characterization of problem-solving competency is implicit and is displaced to identifying  
66 what is meant by a proper physics problem. However, no definition of such problems is given  
67 neither in these texts nor in the reviews of problem-solving in physics (Hsu et al., 2004; Maloney,  
68 1994; Maloney, 2011). There is agreement among physics educators that proper physics problems  
69 are closer to the real world than the traditional and somewhat contrived problems found at the end  
70 of the chapter in textbooks (such as inclined plane problems) (Ding, 2012), but otherwise no  
71 consensus seems to have emerged about the nature of a physics problem: are they well-structured,  
72 ill-structured (e.g, Shekoyan and Etkina 2007) or multifaceted (Ogilvie, 2009), context-rich (Heller  
73 et al. 1992) or context-poor, do they require the making of assumptions about the real world  
74 (Fortus, 2005) etc.?

75           While the characterization of physics problem-solving competency as the ability to tackle  
76 physics problems has much to commend it, including its simplicity and holistic nature, it is too  
77 implicit to inform an understanding of the nature of problem-solving in physics.

78  
79           Some attempts to specify the nature of the problem-solving competency have been offered.  
80 Some of these describe the steps required when solving a problem. Fortus (2007) used the IDEAL-  
81 approach for general problem solving given by Bransden and Stein (1984) where problem solving is  
82 seen as involving the steps 1) Identify the problem, 2) Define and represent the problem, 3) Explore  
83 possible strategies or solutions, 4) Act on a selected solution, 5) Look back and evaluate. Reif  
84 (2008) devised his own general problem-solving strategy (1. Describe problem; 2. Analyze  
85 problem; 3. Construct solution; 4. Assess solution; 5 Exploit the solution). However, these  
86 approaches do not capture the essence of physics problem-solving competency since they use  
87 models for general problem-solving. In contrast to these general approaches, The Quality Assurance  
88 Agency for Higher Education (2017) and Bolton and Ross (1997) focused on *physics* problem-  
89 solving and listed a number of skills that this competency consists of: “For example, students learn  
90 how to identify the appropriate physical principles, how and when to use special and limiting cases  
91 and order-of-magnitude estimates to guide their thinking about a problem and how to present the  
92 solution, making their assumptions and approximations explicit.” (Quality Assurance Agency for  
93 Higher Education, p. 10). Bolton and Ross (1997) added the skill to be able to identify and label  
94 variables. However, while each of these skills may be relevant, simply listing such individual skills  
95 does not constitute an integral approach that is required for the description of a competency, which  
96 is a cluster of related knowledge, skills and attitudes. As the same ingredients can lead to either a  
97 brownie or a chocolate soufflé depending on the sequence of actions, in order to characterize  
98 problem-solving competency we not only need to specify the skills ingredients but also how they  
99 appear in the overall scheme.

100           What is needed is a description of the nature of physics problem-solving that gives a  
101   holistic, explicit characterization. In this article, we use recent historical scholarship to show that  
102   two distinct views on the nature of problem-solving in physics can be discerned among research  
103   physicists in the 20<sup>th</sup> century. The views are epitomized by Arnold Sommerfeld (1868-1951) and  
104   Enrico Fermi (1901-1954), two outstanding and famous physicists of the 20<sup>th</sup> century. The purpose  
105   of the present article is to use their views to characterize and illustrate two positions on the nature of  
106   physics problem-solving competency. The two positions will be called the “theory first,  
107   phenomenon second” position (or “theory first”, for short) and the “phenomenon first, theory  
108   second” positions (or “phenomenon first”, for short), respectively.

109           There are three reasons why we focus on Sommerfeld and Fermi. First, their positions seem  
110   to be characteristic of physicists at various times and places. Among Physics Nobel Laurates, Lev  
111   Landau (1908-1968), Pyotr Kapitza (1894-1984) and Pierre Gilles de Gennes (1932-2007) (see  
112   Livanova, 1978, Kapitza, 1980, Plévert 2011) adhered to something like Fermi’s approach, while  
113   Werner Heisenberg (1901-1976) and Felix Bloch (1905-1983) subscribed to Sommerfeld’s  
114   approach (see Hahn, 1990). Hans Bethe (1906-2005) and John Bardeen (1908-1991) used both  
115   approaches depending on the situation (see Schweber, 2012, and Hoddson and Daitch, 2002). The  
116   second reason is that while one can find proponents of the two approaches that are closer to our  
117   time, the approaches are in fact most elaborately described in the literature on Sommerfeld and  
118   Fermi. The final reason is that Sommerfeld and Fermi not only used their particular problem-  
119   solving approach in their research, but also subscribed to it in their teaching, for instance when they  
120   assigned problems to their students. Hence, for Sommerfeld and Fermi, their teaching reflected their  
121   research practice.<sup>1</sup>

122           The existence and characterization of the two positions are relevant for actors at various  
123   levels. From a policy point of view, the two positions offer quite different perspectives on what

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<sup>1</sup> Lev Landau often used a “phenomenon first” approach in his research, but the problems he assigned in his famous theoretical minimum were oriented towards a “theory-first” approach. Ioffe (2013) gave an example of problem and the famous Landau and Lifschitz textbooks give other problems, which according to Hall (2006), stem from the theoretical minimum.

124 physics can offer for the development of students' problem-solving skills, for instance in relation to  
125 21<sup>st</sup> century skills with its heavy focus on problem-solving (see, e.g., McComas, 2014). From a  
126 learning perspective, as noted above, it is, first, important that the teacher as well as the students are  
127 aware of the nature of the enterprise they are engaged in and hence they should know the nature of  
128 the problem-solving competency they strive to develop; second, the learning environments in which  
129 the two competencies can be developed are most likely different as they require different problems,  
130 scaffolding activities etc. Finally, from a research point of view, the distinction may help to inform  
131 discussions on problem-solving in physics, for instance, about whether a given problem contributes  
132 to one or the other competency.

133         The distinction between the two positions on problem-solving is meant to offer a distinction  
134 between two holistic approaches to the issue of what is physics problem-solving competency in  
135 terms of the relation between theory and phenomenon. We do acknowledge, however, that problem-  
136 solving requires more than identifying the relevant theory and pertinent aspects of the phenomenon.  
137 As noted above, it involves identifying the appropriate physical principles, how and when to use  
138 special and limiting cases and order-of-magnitude estimates, as well as the identification and  
139 labelling of variables and the use of approximations and idealizations. Moreover, it involves  
140 extensive use of mathematics (see Niss (2017)). It is important to keep in mind that these aspects  
141 are part of both approaches.

142

## 143 **2. Two positions on physics problem solving**

### 144 **competency**

145 Both Sommerfeld and Fermi had to invent their own approach to the teaching of theoretical physics.  
146 Sommerfeld because he was a maverick of the budding discipline of theoretical physics, and Fermi  
147 because he was the first professor of theoretical physics in a relatively scientifically isolated Italy.  
148 Sommerfeld's Institute of Theoretical Physics at the University of Munich, which he liked to call "a



149 nursery of theoretical physics,” was very influential and successful and he educated a generation of  
150 German physicists including the Nobel Laurates Wolfgang Pauli, Werner Heisenberg, and Hans  
151 Bethe. Some of Sommerfeld’s students set up branches of the school in Leipzig, Zürich, Stuttgart,  
152 and Hamburg (Eckert 2013). Fermi developed his approach in Rome, then took it to a larger scale in  
153 Chicago, where he trained several outstanding physicists, including the Nobel Laurates Chen-Ning  
154 Yang, T.D. Lee and Jack Steinberger.

## 155 ***2.1 Sommerfeld’s view – The theory-first position***

156 The historian Suman Seth (2010) has characterized the approach of Arnold Sommerfeld (1868-  
157 1951) as “a physics of problems” in contrast with the “physics of principles” advocated by Max  
158 Planck, Albert Einstein and Niels Bohr. The latter searches for general principles, while the former  
159 aims at solving concrete problems. Sommerfeld focused on specific questions and their specific  
160 solutions and searched for a mechanism or a process rather than a generalizing postulate. The  
161 British physicist Frederick Lindemann characterized Sommerfeld’s students in a 1933 letter to  
162 Einstein: “I have the impression that anyone trained by Sommerfeld is the sort of man who can  
163 work out a problem and get an answer, which is what we really need at Oxford, rather than the more  
164 abstract type who would spend his time disputing the philosophers.” (Quoted in Seth 2010, p. 3)  
165 Werner Heisenberg also stressed this focus on problem-solving in Sommerfeld’s teaching: “In his  
166 pedagogy, he was not satisfied with presenting fundamental theoretical relations; rather, he showed  
167 students ‘how it is done,’ how one actually treats a physical problem mathematically through to its  
168 conclusion.” (Heisenberg, 1948), quoted in Eckert 2013, p. 411). In fact, attacking a wide range of  
169 problems mathematically was a characterizing feature of the Sommerfeld’s school (Eckert, 2013).

170 Hans Bethe (1906-2005), who did his PhD with Sommerfeld, has described Sommerfeld’s  
171 approach in more detail: “The method to follow was to set up the differential equation for the  
172 problem (usually the Schrödinger equation), to use your mathematical skill in finding a solution as  
173 accurate and elegant as possible, and then discuss this solution. In the discussion finally, you would

174 find out the qualitative features of the problem. Sommerfeld's way was a good one for many  
175 problems where the fundamental physics was already understood, but it was extremely laborious. It  
176 would usually take several months before you knew the answer to the question." (Segrè 1970, p. 59)  
177 The differential equation for the problem did need not to be the Schrödinger equation, but could  
178 come from other areas of physics, including electromagnetism or classical mechanics.

179         So, Sommerfeld advocated an approach that was mathematically as well as physically  
180 rigorous. The starting point is a problem that is defined within the framework of a theory of physics.  
181 Hence, the first major task for the problem-solver is to adapt the equation or principle correctly to  
182 the situation; to do this requires introducing approximations and idealizations, identifying the  
183 relevant variables, boundary conditions and constraints. The next step is to set up the equation  
184 within that framework. Then the equation is solved and finally the solution is discussed in terms of  
185 the relevant physical theory. In short, it was a "theory first, phenomenon second" approach in the  
186 sense that the problems concern whatever phenomena that arise within the theory. It is clear from  
187 the theory what equation(s) is (are) relevant and the task of the problem-solver is mainly to solve  
188 the mathematical problems involved.

189         Sommerfeld's teaching reflects these research ideals, e.g., for the problems he suggested to  
190 his students. One student, Paul Ewald, recalled that Sommerfeld had a list of a doctoral thesis  
191 topics, such as the calculation of self-inductances of solenoids for alternating currents, the  
192 propagation of radio waves over a surface of finite conductivity or an unsolved problem of  
193 gyroscopic theory (Eckert 2013). Each subject had its own merit and its own type of mathematical  
194 technique that were pointed out by Sommerfeld.

195         Sommerfeld's position can be described as the "theory-first" position on physics problem-  
196 solving competency. The problems to be solved originate in and are defined by one of the theories  
197 of physics, be it quantum mechanics, classical mechanics, electromagnetism etc. The problems  
198 concern physical phenomena, but these phenomena arise within the theory rather than the other way  
199 around, that is observe some phenomenon that we try to understanding using whatever available

200 theory. This implies that not only the physical theory is given, but typically the relevant equation or  
201 principle of that theory is also given, so the identification of the theory and the equation/principle  
202 play a minor role in the problem-solving process. Now the skills mentioned in the introduction  
203 come into play. The next step is to make some idealizations and abstractions concerning the  
204 physical system in question, such as assuming perfectly spherical objects and neglect air resistance.  
205 Then the problem-solver needs to identify the relevant variables, boundary conditions and  
206 constraints. Next the problem-solver has to adapt the equation or principle correctly to the situation.  
207 This may be quite demanding on the part of the problem-solver. Next, the problem-solver should  
208 apply her mathematical skills and solve the mathematical problem “as accurately and elegantly” as  
209 possible. Of course, this could be quite demanding. Finally, the problem-solver should interpret and  
210 discuss the solution in terms of the original problem situation, as well as consider the qualitative  
211 features of the solution.

## 212 ***2.2 Fermi’s view – The phenomenon-first position***

213 The physics approach of Enrico Fermi (1901-1954) can, like Sommerfeld’s, be characterized as a  
214 physics of problems, as he preferred to work on concrete problems rather than study abstract and  
215 general principles. C. N. Yang recalled about Fermi’s teaching that “We learned that abstractions  
216 come *after* detailed foundation work, not before.” (Segrè 1970, p. 170). However, while  
217 Sommerfeld favored a very theoretical approach, Fermi tried to make the physics of a problem clear  
218 and he often gave beautiful, simple and clear explanations of puzzling phenomena (Segrè 1970).  
219 Hans Bethe, who worked with both Sommerfeld and Fermi, agreed and called Fermi’s approach  
220 “pragmatic” (Segrè, 1970, p. 60). Fermi would solve a problem by thinking about it in a general  
221 way, making an analysis of the essentials and providing a few order-of-magnitude estimates: “He  
222 was able to analyze into its essentials every problem, however complicated it seemed to be. He  
223 stripped it of mathematical complications and of unnecessary formalism. In this way, often in half  
224 an hour or less, he could solve the essential physical problem involved. Of course, there was not yet  
225 a mathematically complete solution, but when you left Fermi after one of these discussions, it was

226 clear how the mathematical solution should proceed.” (Segrè 1970, p. 59). Fermi’s was a  
227 “phenomenon first, theory second” approach, in the sense that it first involved an analysis of the  
228 phenomenon, then a description of it within a theory of physics. Here, analysis of the phenomenon  
229 means an explanation of the phenomenon. “Often, when just talking to [Fermi], one heard a  
230 beautiful explanation develop, simple and clear, which would resolve a puzzling phenomenon.”  
231 (Segrè, 1970, p. 54). Such an explanation is based on physical concepts and principles and gives a  
232 physical narrative of the phenomenon, and hence “resolves” the phenomenon in the physics and the  
233 essential physics had been found. An important check that the explanation resolves the phenomenon  
234 is the use of order-of-magnitude estimates based on the explanation to see whether the explanation  
235 gives roughly the right numbers.

236         The mathematics was always subordinate to the physics for Fermi and he chose the  
237 mathematical tools for the occasion. Personally, Fermi was not intimidated by mathematical  
238 difficulty, but he did not seek elegant mathematics for its own sake: “the important point is whether  
239 it illuminates the essential physical content of the situation.” (Segrè 1970, p. 18). Bethe has given  
240 this account of the difference between Sommerfeld and Fermi: “Sommerfeld said ‘Well here is the  
241 title of your problem, now you do it and then you had to put in differential equations and if possible  
242 Bessel functions. For Fermi that didn’t matter. You just did the mathematics the best way that came  
243 to your mind, and the physics was clear by the time you started.” (Schweber 2012, p. 195)

244         Like Sommerfeld, Fermi practiced what he preached, that is he tried to develop the same  
245 problem-solving skills in his students that he also used in his research as testified in the collection  
246 of problems that he used at University of Chicago and which led to (Cronin, Greenberg and Telegdi,  
247 1979).

248         Fermi’s position can be described as the “phenomenon-first” position in problem-solving in  
249 physics.<sup>2</sup> The starting point of problem-solving is some phenomenon in the physical world. The  
250 first task of the problem-solver is to make a qualitative analysis of the phenomenon and describe the

---

<sup>2</sup> Fermi is the inventor of Fermi problems. However, one should note that while the above approach in general accords with Fermi problems, the latter, such as the famous number of Piano Tuners in Chicago, often do not require physical reasoning, in contrast to the crucial steps involving physics of the above approach.

251 essentials of the situation by giving an explanation of the phenomenon in terms of physical concepts  
252 and principles. This should lead to an identification of the relevant physical theory and the relevant  
253 equations. This step could be a quite substantial part of the solution process. Then the problem-  
254 solver uses this identification of the physics to describe the situation and convert the problem into a  
255 model susceptible to quantitative analysis by extracting essential elements and idealizing these  
256 elements if necessary; this includes comparing effects by making rough order-of-magnitude  
257 calculations. Next, the problem-solver should use whatever mathematics might be deemed relevant  
258 for analyzing the model. The problem-solver then conducts a mathematical analysis, by making the  
259 necessary approximations and using the relevant information. Finally, the mathematical results  
260 obtained are interpreted in terms of the real-world situation and an answer to the specific question is  
261 given.

262         The two positions differ substantially on a number of issues. First, there is a difference when  
263 it comes to the required physical analysis. In the phenomenon-first approach, a major task is to give  
264 a qualitative analysis of the situation, whereas this grows out of the solution for the theory-first  
265 approach. Second, the role of theory is different. In the theory-first approach, the problem originates  
266 in the theory and is defined by it, whereas according to the phenomenon-first approach, the theory is  
267 a framework that delivers tools that can be used for a particular problem. Finally, the interpretation  
268 of the mathematical result in terms of the situation differs. For the phenomenon-first approach, the  
269 result answers a specific question, whereas for the other approach the result gives general insight  
270 into the behavior of the problem.

271

### 272 **3. Illustration of the two positions**

273 The two positions can be illustrated by looking at the solutions to the following two problems that  
274 come from problem collections used in physics education.

275

- **Problem 1: The spiral orbit problem.** A particle moves in two dimensions under the influence of a central force determined by the potential  $V(r) = \alpha r^p + \beta r^q$ . Find the powers  $p$  and  $q$  which make it possible to achieve a spiral orbit of the form  $r = c\theta^2$ , with  $c$  a constant. (Cahn et al. 1994, p. 10)
- **Problem 2: The pole vaulting problem.** When told that the world record for the pole vault was about 18 feet, the fast-rising athlete Rod Fibreglass told the press, “Give me a pole long enough, and I will raise the record to 30 feet”. Could he manage it? How high might he get if he tried hard? (Thompson 1987, p. 3)

The first problem is couched in technical language and may look daunting to an outsider with its reference to central forces and mathematical equation for the potential. It is clear that this technical information is required to solve the problem and the initiated reader will immediately understand that this is a problem in classical mechanics, but perhaps not know how to solve it within that theoretical framework. The other problem, on the other hand, can be immediately understood by the non-initiated reader. Here it is not clear, however, what physics is relevant to the problem or how to apply the theories of physics to it, perhaps even for a reader well-versed in physics.

### **3.1 The solution to the spiral orbit problem and the theory-first position**

The following solution to problem 1 is a sketch of the solution given in the textbook Cahn & Nadgorny (1994). They start by pointing out that “The solution may be obtained most quickly by employing the differential equation for the orbit (see Goldstein, *Classical Mechanics*, §3-5)” (Cahn et al. 1994, p.100). Section §3-5 of this classical textbook is entitled “The differential Equation for the Orbit, and Integrable Power-Law Potentials” and is found in the chapter on the two-body central force problem, which applies to systems where two bodies interact with a force that acts only in the direction of the line connecting the two bodies. In the beginning of the chapter, Goldstein shows

301 that for such systems the so-called angular momentum vector is conserved and he sets up the  
 302 fundamental differential equations equivalent to Newton's second law for the situations in question.  
 303 The relevant equation for our purposes, which the reader may wish to simply accept, is

$$304 \quad m \frac{d^2 r}{dt^2} - \frac{l^2}{mr^3} = - \frac{\partial V}{\partial r}. \quad (1)$$

305 Here  $r$  is the distance,  $m$  is the mass,  $l$  is the magnitude of the angular momentum.  $\frac{\partial V}{\partial r}$  is the  
 306 derivative of the potential  $V(r)$ , which is a measure of the force between the particles.

307

308 Cahn et al. (1994) now states that problem 1 can be solved by applying Goldstein's line of  
 309 reasoning in section §3-5 where he eliminates the time dependence from the above equation using  
 310 the conservation of angular momentum. This leads to following differential equation

311

$$312 \quad \frac{d^2 u}{d\theta^2} + u = - \frac{m}{l^2} \frac{d}{du} V \left( \frac{1}{u} \right). \quad (2)$$

313

314 Here  $u$  is defined by  $u = \frac{1}{r}$ .

315

316 We can now substitute the problem's orbit equation  $r = c\theta^2$  into the definition of  $u$  to get

$$317 \quad u = \frac{1}{r} = \frac{1}{c\theta^2} \text{ so } \frac{d^2 u}{d\theta^2} = \frac{6}{c\theta^4}.$$

318

319 Substituting this result and the problem's potential equation  $(V(r) = \alpha r^p + \beta r^q = \alpha \left(\frac{1}{u}\right)^p +$

320  $\beta \left(\frac{1}{u}\right)^q)$  into equation 2 yields

321

$$322 \quad \frac{6}{c\theta^4} + \frac{1}{c\theta^2} = \frac{m}{l^2} (p\alpha c^{p+1}\theta^{2(p+1)} + q\beta c^{q+1}\theta^{2(q+1)}) . \quad (3)$$

323

324 Using that so far  $p$  and  $q$  are interchangeable, we can identify powers of  $\theta$  on the two sides of  
325 equation 3 to obtain

326 
$$-4 = 2(p + 1)$$

327 
$$-2 = 2(q + 1)$$

328

329 and therefore we get the necessary condition

330 
$$p = -3$$

331 
$$q = -2$$

332 (Cahn et al. 1994, p.100-101).

333 This solution illustrates the theory-first position. At first, the problem-solver should identify  
334 the relevant governing equation from the available ones; in this case it is the equations of classical  
335 mechanics. The problem-solver can make this identification by knowing equation 2 from the  
336 literature, say the Goldstein reference, or by deriving it herself using the outlined procedure.<sup>3</sup> Next,  
337 the problem-solver adapts this equation to the present situation, namely the given potential and the  
338 given equation for a spiral orbit in equation 2. Then the job is mainly mathematical, which is to  
339 rearrange this equation and identify powers on the two sides. One could then (but this is not done in  
340 the solution given in the book), give answers to qualitative questions, such as what physical system  
341 corresponds to the potential obtained, i.e.

342

343 
$$V(r) = \alpha r^{-3} + \beta r^{-2}.$$

344

345 or how do  $a$ ,  $b$ , and  $c$  relate to conserved angular momentum in a particular motion?

346 In the case of the spiral orbit problem, the initial physical analysis consists in saying that  
347 since we have central force motion (as described in the problem text), angular momentum is  
348 conserved, so we can use the equation from Goldstein. The fact that angular momentum is

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<sup>3</sup> The procedure is to use the definition of angular momentum  $l$ , which is conserved in central force motion, and substitute  $\left(\frac{l}{mr^2}\right)\left(\frac{d}{d\theta}\right)$  for  $\frac{d}{dt}$ .



349 conserved is standard reasoning in classical mechanics. The problem provides the potential as well  
350 as the orbit equation, so it neither requires that the problem-solver contemplates realistic potentials  
351 nor that she sets up the condition for spiral orbits. Basically, the analysis of the physics of the  
352 situation comes after the problem has been solved, namely answering qualitative answers such as  
353 the two questions mentioned above.

### 354 **3.2 The solution to problem 2 and the phenomenon-first position**

355 Turning now to problem 2, Thompson (1987) solved the problem in the following way. First, it  
356 should be realized that the dominant consideration is that the kinetic energy of the running man just  
357 before take-off is converted into gravitational potential energy during the jump at something less  
358 than 100% efficiency. This reasoning gives the equation

$$359 \quad \frac{1}{2} mv^2 = mgh.$$

360 Here  $v$  is the speed of the running man,  $h$  is the height the man can reach,  $m$  is his mass and  $g$  is the  
361 gravitational acceleration. We have to isolate  $h$ , and get

$$362 \quad h = \frac{1}{2} \frac{v^2}{g}.$$

363 We have to somehow estimate the speed of the pole vaulter during the run. The world record in 100  
364 meters race is about 10 s, giving an average speed of  $10 \text{ m s}^{-1}$ . If the pole vaulter attains a speed of  
365  $10 \text{ m s}^{-1}$  during the run, the corresponding height is

$$366 \quad h = \frac{1}{2} \frac{\left(\frac{10\text{m}}{\text{s}}\right)^2}{9.82\text{m/s}^2} \approx 5\text{m}$$

367 To this number, we should add smaller terms arising from: (i) the fact that his center of mass is  
368 already about 1 m above the ground when he starts; (ii) the work done by his legs on take-off, and  
369 by his arms in climbing up the pole, which can be estimate to give an extra 0.5m; (iii) the fact that  
370 his center of mass actually passes *below* the bar, an extra 10cm. Adding these items together gives  
371 an answer of approximately:

$$H = \frac{1}{2} \frac{v^2}{g} + 1m + 0.5m + 0.1m = 6.6m = 21 \text{ feet.}$$

The difference between this and the observed 18 feet is due to an efficiency of less than 100% – or to errors in the estimated quantities. In any case, there is clearly no hope of Mr Fibreglass making good his boast that he could raise the record to 30 feet as stated in the problem formulation.

This problem illustrates Fermi’s approach. First, the problem-solver has to make a physical analysis of the situation. She should realize that the dominant consideration is that the problem can be solved using the principle of the conservation of energy and that she then has to equate the kinetic energy during the running phase of the pole vaulting with the potential energy during the jump, as this will give an upper bound on the height the pole vaulter can reach. So, she has to recognize that the problem doesn’t require that the specifics of the situation be taken into account, such as the elastic properties of the rod and also that the mentioned terms (the center-of-mass of the body is above the ground, the work of the feet, the center of mass of the body is going under the stick) are in fact not the dominant ones. With these considerations in place, the mathematical solution is not very complicated involved and the problem solver only has to isolate  $h$ . Next, the problem-solver has to realize the presence of the other terms and estimate their values – based on her physical understanding of the situation. Finally, the result is interpreted in terms of the original question of whether it is possible to jump 30 feet up.

## 4. Remarks on the two approaches

The theory-first and the phenomenon-first approaches emphasize different sides of the problem-solving competency: the initial physical analysis, as well as the theory and the interpretation of the mathematical results play different roles in these two understandings.

The two approaches are not contradictory; in fact, the phenomenon-first approach is often seen as a sort of supplement to the theory-first approach. In the 1960s, the physics department at the

396 University of Bristol, for instance, introduced an exam using problems like the pole vault one in  
397 order to see whether the student could use the material previously learned in courses with more  
398 traditional exam problems such as the spiral orbit problem (Thompson 1987). Moreover, it was  
399 hoped that the exam could encourage the cultivation of a group of skills considered as an important  
400 constituent of the expertise of the professional physicist, including the ability to convert a real  
401 problem into a model. The book is entitled “Thinking like a physicist”, to stress the importance of  
402 these skills, which are not trained by solving the more traditional exam problems.

403         The Nobel Laureate Pyotr Kapitza (1894-1984) had a similar agenda and advocated using  
404 problems related to Bristol’s problems in his general physics course in the 1940s. Kapitza found  
405 that the problems could cultivate the creative scientific thinking of future scientists, as it “is well-  
406 known that fruitful scientific work requires not only knowledge and understanding but also a  
407 capacity for independent analytical and creative thinking. In effect, these problems were compiled  
408 as a useful means for the discovery, evaluation and cultivation of these qualities during the teaching  
409 process.” (Kapitza 1980, p. 198) Some physicists combined both approaches in their research; this  
410 includes Hans Bethe, whose craftsmanship as a physicist has been described by his biographer as  
411 “an amalgam of what he learned from these two great physicists and teachers [Sommerfeld and  
412 Fermi], combining the best of both: The thoroughness and rigor of Sommerfeld with the clarity and  
413 simplicity of Fermi. He had learned from them how to balance rigorous analysis with approximate  
414 methods.” (Schweber 2013, p. 196) This craftsmanship meant that Bethe, in the words of Freeman  
415 Dyson, was “the supreme problem solver of the past century.” (Dyson 2005, p. 219) In short, the  
416 two understandings of physics problem-solving competency are two co-existing approaches rather  
417 than two mutually exclusive ones.

418

## 419 **5. Concluding remarks**

420 Given that physics students are to acquire problem-solving competency during their physics  
421 education, educators need to make decisions about the kind of problem-solving competency. In this  
422 article, we have presented two different views on physics problem solving, the theory-first and the  
423 phenomenon-first approaches. Historical scholarship has shown that these are characteristic for the  
424 university level teaching of two prominent physicists of the 20<sup>th</sup> century. As shown above, the two  
425 positions can be found in other parts of the university level physics education community as well,  
426 for instance when it comes to the selection of physics problems at college/university level. A further  
427 example is a problem-solving course in physics at Roskilde University (Niss & Højgaard Jensen,  
428 2010). The course is based on solving the so-called unformalized problems in physics, where  
429 problems are formulated in everyday language and often concern real world phenomena (Højgaard  
430 Jensen, Niss & Jankvist, 2017). Consequently, a major aspect of the problem-solving process is to  
431 formalize the problem in physics terms. Hence, these problem sits squarely within the Fermi  
432 tradition and a similar view has informed the development of the course (Niss & Højgaard Jensen,  
433 2010).

434 The two positions seem to have something to offer for educational discussions on problem-  
435 solving at college/university level physics. An example from Danish Highschool indicates that the  
436 two positions are also relevant for discussion for physics education at lower levels that presumably  
437 are farther removed from research physics. The problems at the written exam at the highest level of  
438 physics in the Danish high school have traditionally been of the theory-first kind. However, within  
439 the last ten years or so, the problem sets at the exams have begun to be supplemented by problems  
440 that follow more the phenomenon-first approach as the problem-solver now has to make a thorough  
441 physical analysis of the problem situation. So, today both views on problem-solving competency is  
442 at play in the exam sets in Danish Highschool. An awareness of the two positions might clarify for  
443 teachers and students alike that actually two fundamentally different views are at play in the exam  
444 problems.

445 Finally, it should be pointed out that the two versions of the problem-solving competency  
446 need to be taught differently. Different kinds of problems are needed, depending on which  
447 competency we are talking about, but it is reasonable to assume that the two competencies also  
448 require different scaffolding activities as the solution processes require different planning,  
449 monitoring and justifications. While some of the extensive research that has been done on problem  
450 solving in physics might be relevant, it seems that further works need to be done in this respect.

451

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## Response to comments

The manuscript has been revised according to the comments given directly in the manuscript.